RESEARCH ARTICLE OPEN ACCESS

Total Negative Edge of Helm Graphs

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Abstract:

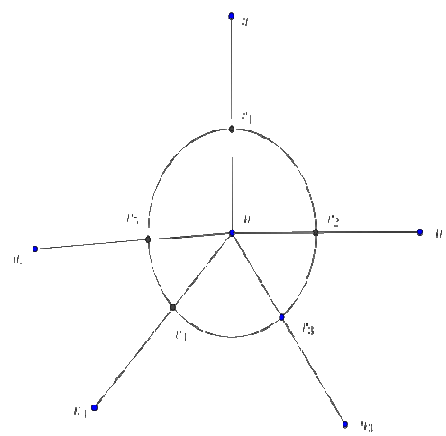
The Total Vertex Irregularity Strength, tvs(G), is the minimum value of the largest label over all such irregular assignments. It is interesting to see that addition of an edge from the complement of the graph to the graph G may increase or decrease the total vertex irregularity strength of G or remains the same. Thus we call it as total positive edge, total negative edge and total stable edge of G respectively.

INTRODUCTION

Let $G = (V, E)$ be a simple graph. By a total labeling of a graph we will mean an assignment $f: E \cup V \to Z^+$ to the edges and vertices of G. The weight of a vertex $v \in V$, is defined by $w(v) = f(v) + \sum_{vu \in E} f(vu)$. Moreover, the weighting f is called irregular if for each pair of different vertices their weights are distinct. The total vertex irregularity strength, $tvs(G)$, is the minimum value of the largest label over all such irregular assignments. In [2], Martin Baca et al., determined the total vertex irregularity strength of complete graphs, prisms and star graphs.

A **Helm graph** H_n is the graph obtained from a wheel by attaching a pendant edge at each vertex of the *n*-cycle. The vertex set of H_n is $V = \{u, v_i, u_i : 1 \le i \le n\}$ and the edge set of H_n is $E = \{uv_i,\ v_iv_{i+1},\ v_iu_i : 1 \le i \le n\}$, with indices taken modulo *n*.

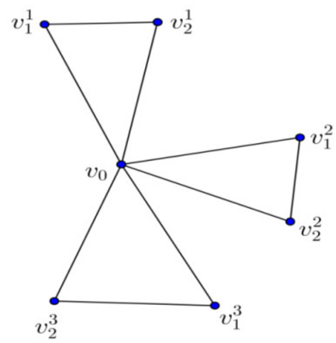
Example



Helm graph H_5

A **generalised friendship graph** $f_{m,n}$ is a collection of *m* cycles of order *n* meeting in a common vertex. The vertex set of $f_{m,n}$ is $V = \{v_j^i : 1 \le i \le n\}$ and the edge set of $f_{m,n}$ is $E = \{v_j^i v_{j+1}^i : 1 \le i \le m \cap 0 \le j \le n-1\}$, with indices taken modulo *n*.

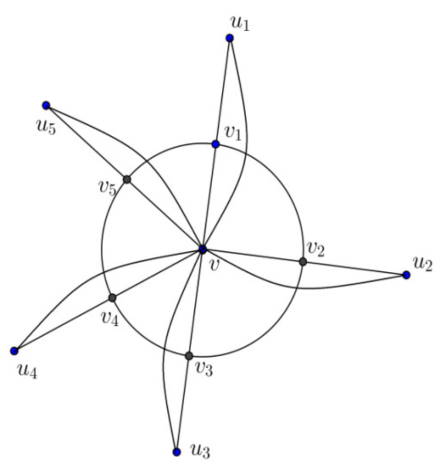
Example



Friendship graph $f_{3,6}$

A **flower graph F_n** is the graph obtained from a helm by joining each pendant vertex to the helm. The vertex set of F_n is $V = \{v, v_i, u_i : 1 \leq i \leq n\}$ and the edge set of F_n is $E = \{vv_i, vu_i, v_iv_{i+1}, v_iu_i : 1 \leq i \leq n\}$, with indices taken modulo *n.*

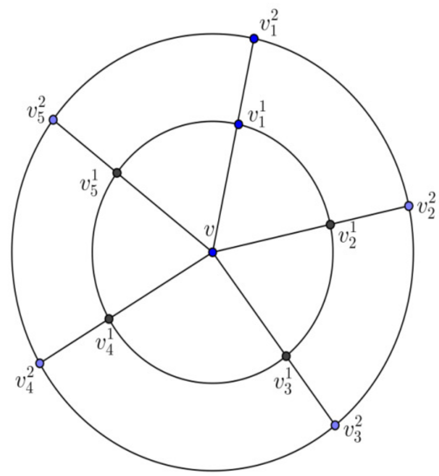
Example



Flower Graph F_5

A **web graph Wb_n** is the graph obtained from a helm by joining the pendant vertices to form an *n*-cycle. The vertex set of Wb_n is $V = \{v, v_j^i : 1 \leq i \leq 2 \cap 1 \leq j \leq n\}$, and the edge set of Wb_n is $E = \{vv_j^1, v_j^1v_j^2, v_j^1v_{j+1}^1, v_j^2v_{j+1}^2 : 1 \leq j \leq n\}$, with indices taken modulo *n.*

Example



Web Graph Wb_5

Theorem

For $n \geq 4$, the total vertex irregularity strength of H_n is

$$\text{tvs}(H_n) = \left\lceil \frac{n+1}{2} \right\rceil$$

Proof.

The vertex set and edge set of the helm H_n are

$V(H_n) = \{u_i; v_i : 1 \leq i \leq n\} \cup \{u\}$
$E(H_n) = \{v_iv_{i+1}; u_iv_i ; uv_i : 1 \leq i \leq n\}$

Consider the vertices of degree 1. There are *n* such vertices, and if we want to use only the labels 1,2,…,*s*, the lowest and highest weights that we can obtain are respectively 2 and 2*s*, which implies that $2s - 2 + 1 \geq n$ and thus

$$\text{tvs}(H_n) \geq \left\lceil \frac{n+1}{2} \right\rceil$$

To show that $tvs(H_n) \leq \left\lceil \frac{n+1}{2} \right\rceil$, we define a labeling $\emptyset : V(H_n) \cup E(H_n) \to \{1, 2, \ldots, \left\lceil \frac{n+1}{2} \right\rceil\}$ as follows:

$$\emptyset(v_iv_{i+1}) = \emptyset(u) = \emptyset(uv_i) = \left\lceil \frac{n+1}{2} \right\rceil,$$

for $1 \leq i \leq n$

$$\emptyset(v_i) = \emptyset(u_i) = \begin{cases} 1, & \text{for } i = 1 \\ 2, & \text{for } 2 \le i \le 4 \\ \left\lceil \frac{i}{2} \right\rceil, & \text{for } 5 \le i \le n \end{cases}$$

$$\emptyset(v_i u_i) = \begin{cases} 1, & \text{for } i = 1,2 \\ 2, & \text{for } i = 3 \\ \left\lceil \frac{i+1}{2} \right\rceil, & \text{for } 4 \le i \le n \end{cases}$$

This labeling gives the weight of the vertices, u, u_i and v_i for $1 \le i \le n$, as follows:

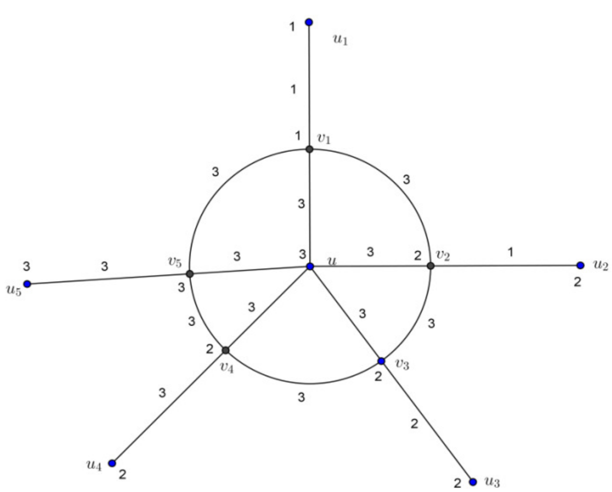
$$wt(u) = \left\lceil \frac{n+1}{2} \right\rceil (n+1);$$

$$wt(u_i) = i + 1;$$

$$wt(v_i) = 3\left\lceil \frac{n+1}{2} \right\rceil + 1 + i.$$

The weight of the vertices are distinct, and thus $\emptyset$ is the vertex irregular total labeling of the helm graph H_n.

Example



tvs $(H_5) = \left\lceil \frac{5+1}{2} \right\rceil = 3$

Let $G = (V, E)$ be any graph which is not complete. Let e be any edge of $\bar{G}$, then e is called a **total positive edge of G,** if $tvs(G+e) > tvs(G)$. The **total negative edge** and **total stable edge** of G if $tvs(G+e) < tvs(G)$ and $tvs(G+e) = tvs(G)$ respectively. If joining of any two non-adjacent vertices of G, by an edge increases the total vertex irregularity strength of G, then G is a total positive graph. If it decreases the total vertex irregularity strength of G, then G is a total negative graph.

II MAIN RESULT

In this section, we study total negative edge of helm graphs (H_n) for $n \equiv 2 (mod\ 4)$ and $n \equiv 3 (mod\ 4)$.

Theorem

For $n \equiv 2 (mod\ 4)$, the Helm graph n ≥ 4 is a total negative graph.

Proof.

The vertex set and edge set of H_n are

$$V(H_n) = \{u_i, v_i : 1 \le i \le n\} \cup \{u\}$$

$$E(H_n) = \{v_i v_{i+1}, u_i v_i, u v_i, u_{n-1} u_n : 1 \le i \le n\}$$

Add the edge $u_{n-1}u_n$ to H_n, then define the total labeling, φ as follows:

(i) $\varphi(u) = 1$

(ii) $\varphi(v_i) = \begin{cases} \frac{n-2}{4}; & i = 1 \\ \frac{n}{2}; & 2 \le i \le n \end{cases}$

(iii) $\varphi(u_i) = \begin{cases} 1; & i = 1 \\ 2; & 2 \le i \le \frac{n}{2} + 1 \\ i + 1 - \frac{n}{2}; & \frac{n}{2} + 2 \le i \le n - 2 \\ \frac{n}{2} - 1; & i = n - 1 \\ \frac{n}{2}; & i = n \end{cases}$

(iv) $\varphi(u_i v_i) = \begin{cases} 1; & i = 1 \\ i - 1; & 2 \le i \le \frac{n}{2} \\ \frac{n}{2}; & \frac{n}{2} + 1 \le i \le n \end{cases}$

(v) $\varphi(v_i v_{i+1}) = 2; \quad 1 \le i \le \frac{n}{2}$

(vi) $\varphi(v_i v_{i+1}) = \begin{cases} \frac{i+1}{2}; & i \text{ is odd} \\ \frac{i}{2}; & i \text{ is even} \end{cases}; \quad \frac{n}{2} < i \le n$

(vii) $\varphi(u v_i) = \frac{n}{2}; \ \forall i$

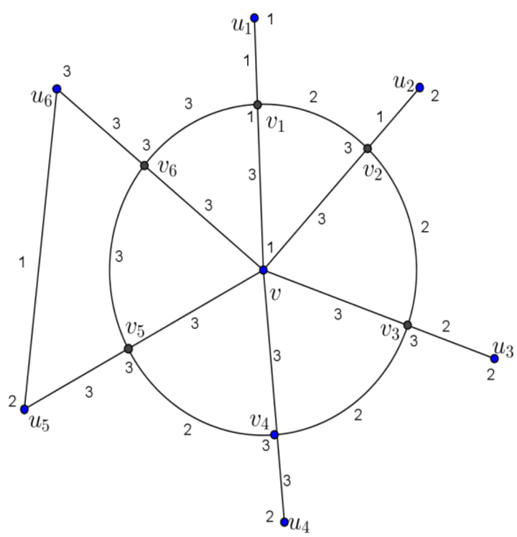
(viii) $\varphi(u_{n-1} u_n) = 1$

By the above irregularity total labeling we get,

$$tvs(H_n + u_{n-1}u_n) \leq \frac{n}{2} < tvs(H_n)$$

Thus, $u_{n-1}u_n$ is a total negative edge. Since $H_n + u_iu_{i+1} \cong H_n + u_{n-1}u_n$, $1 \leq i \leq n$, u_iu_{i+1} is a total negative edges for all i.

Example



$tvs\ (H_6 + u_5u_6) = 3$

Theorem

For $n \equiv 3(mod\ 4)$, the Helm graph $H_n, n \geq 4$ is a total negative graph.

Proof.

The vertex set and edge set of H_n be

$V(H_n) = \{u_i, v_i : 1 \leq i \leq n\} \cup \{u\}$

$E(H_n) = \{v_iv_{i+1}, u_iv_i, uv_i, u_{n-1}u_n : 1 \leq i \leq n\}$

Add the edge $u_{n-1}u_n$ to H_n and then define the total labeling, φ as follows:

(i) φ

(ii) $\varphi(v_i) = \begin{cases} \frac{n-3}{4}; & \\ \left\lfloor \frac{n}{2} \right\rfloor; & 2 \end{cases}$

(iii) $\varphi(u_i) =$

$$\begin{cases} i; & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ \left\lfloor \frac{n}{2} \right\rfloor; & \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n, i \neq n-1 \\ \frac{n}{2}; & i = n-1 \end{cases}$$

(iv) $\varphi(u_iv_i) =$

$$\begin{cases} 1; & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ i + 1 - \left\lfloor \frac{n}{2} \right\rfloor; & \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n \\ \left\lfloor \frac{n}{2} \right\rfloor; & n-2 \leq i \leq n \end{cases}$$

(v) $\varphi(v_1v$

(vi) $\varphi(v_iv_{i+1}) =$

$$\begin{cases} \frac{n-3}{4} + \frac{i}{2}; & i \text{ is even} \\ \frac{n-3}{4} + \frac{i-1}{2}; & i \text{ is odd} \end{cases}; \quad 22 \leq$$

$2 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor - 1,$

(vii) $\varphi(v$ $\left\lfloor \frac{n}{2} \right\rfloor - 1;\ \left\lfloor \frac{n}{2} \right\rfloor \leq i$

(viii) $\varphi(v_iv_{i+1}) = \left\lfloor \frac{n}{2} \right\rfloor;\ i = n -$

$1, n$

(ix) $\varphi(uv_i) =$

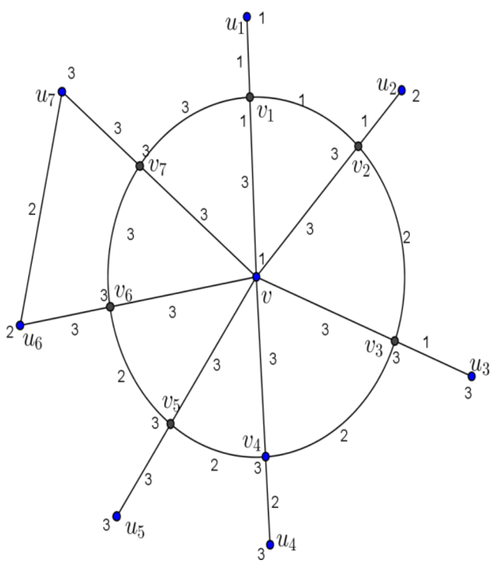
(x) $\varphi(u_{n-}$

By the above irregularity total labeling we get,

$$tvs(H_n + u_{n-1}u_n) \leq \left\lfloor \frac{n}{2} \right\rfloor < tvs(H_n)$$

Thus, $u_{n-1}u_n$ is a total negative edge. Since $H_n + u_iu_{i+1} \cong H_n + u_{n-1}u_n, 1 \leq i \leq n, u_iu_{i+1}$ is a total negative edges for all i.

Example



$tvs(H_7 + u_6u_7) = 3$

Conclusion

In this paper we proved that joining of any two consecutive pendant vertices in helm graph (H_n) $n \geq 4$, is a total negative edge for $n \equiv 2(mod\ 4)$ and $n \equiv 3(mod\ 4)$. Thus helm graph (H_n) $n \geq 4$, is a total negative graph for $n \equiv 2(mod\ 4)$ and $n \equiv 3(mod\ 4)$.

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