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Constructing of Cubic Bezier Interpolation Curve Based on Curvature Constraint

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Abstract:

For today's curve design,In view of the fact that curve fairing is not guaranteed in curve construction, a curvature constraint based smoothing interpolation curve construction method is proposed, which ensures the smoothness of reconstructed curves. Therefore, in view of this problem, this paper analyzes the curve function of the spline, and aims at the law of curvature change.With the minimum absolute value of the derivative of curvatureas the constraint,a setof equationsisformed with the curve function. The Gauss-Legendre quadrature formula is applied to deal with the equation. Finally,the optimal solution is obtained by the block coordinate descent method,and the method of curvature fairing curve design is obtained.

Keywords — **curvature constraint,Bézier interpolation curve,optimal solution,curve fairing**

1.INTRODUCTION

In recent years, with the rapid development of computer technology, computer aided design CAD (Computer Aided Design) has been closely related to modern manufacturing industry. With the aid of various auxiliary design software (such as Imageware, UG, Pro-E, etc.), the design and correction of the shape of the product has become an indispensable link in the design and production of modern manufacturing industry. In the process of product shape design, the curve and surface of the product are the most important factors, not only for the needs of some special industry products (such as cars, space, etc.). Good curves and surfaces are also applied to consumer products, such as toothbrushes, cell phones, health equipment, etc. In order to get higher order continuous and fairing surfaces, the high quality curves that form the surface become the first step for design engineers. Among them, we can use constraints to smooth curves, Yong and so on[1],A class of three degree G1 interpolation curves called optimal geometric Hermite is proposed, which can get the fairing three degree curves by determining the optimal value of the tangent length of the end points. Lu[2],A five time G2 interpolation curve that can handle arbitrary G2 endpoint conditions is proposed. But the comparison and use of constraints can be used to smooth the curve. It is more appropriate and more intuitive to describe the smoothness of the curve from a mathematical point of view with the change of curvature. The smoothing curve is transformed into a mathematical constraint condition, and the important one is the smooth curve with relatively smooth curvature[3].In this paper, cubic Bézier curve is used as an expression to study the construction method of interpolation curve[4]based on with minimum curvature change rate.By constructing three Bézier curve curve with curvature variation equations as constraint conditions, The Gauss-Legendre formula is used to deal with the equation. Finally, the method of solving the smooth Bézier curve is obtained by using the block coordinate descent method to obtain the optimal solution. Experimental verification shows that the curvature change rate of the curve obtained by this method is the smallest, that is, the curve is smooth.

2.Curve fairing criterion

Before studying the curve smoothing algorithm, we first put forward a fairing criterion. The smooth, literal meaning is smooth and smooth. If a curve has many convex and concave point or greater curvature, then intuitively think this curve is not smooth. In general, it is considered that in all curves interpolated to a fixed value point, the elastic spline is the most smooth by giving the fixed value points. However, it is difficult to give precise definition of the smoothness from a rigorous scientific induction. Because the smoothness involves the aesthetic appearance of the geometric appearance, it has a certain subjectivity and has different fairing requirements for the specific objects. Although the observation and definition of fairing is subjective, the definition of fairing criterion is not the same in different literatures[5-8],However, the smoothness of the curve also has some objectivity, and there are some basic requirements to be reflected in the construction of the curve of fairing, for example, "the curvature change is more uniform". Su,Liu and so on[9]The fairing criteria given in 1981: 1) the two order parameter is continuous. Continuous); 2) no excess inflection points; 3) the curvature changes are more uniform. Shi and so on giving the fairing criterion[10]1) two order geometric continuity (location, tangent direction and curvature vector continuity, referred to as continuous curvature). 2) there are no singularities and redundant inflection points; 3) curvature changes are relatively small; 4) strain energy is small. Farin[11]In 1988, it was proposed that a curve is smooth, if the corresponding curvature curve is continuous, (for the relative curvature of a plane curve), there is an appropriate symbol (if the concave and convexity of the curve is known), as close as possible to a monotone function, and the monotone segment of the monotone function is as small as possible. Therefore, it is advisable to use curvature change as the smoothing condition of observation curve and use it as a constraint to construct fairing curve.

3 Cubic Bézier interpolation curve and curvature constrained mathematical expression

Given initial conditions $\{P_0, P_1, T_0, T_1\}$, P_0 P_1 them represent the starting point and end point, T_0 T_1 that unit tangent vector that represents the initial two points, respectively. Given φ_0 for T_0 of P_0P_1 included angle,Given φ_1 for P_0P_1 of T_1 included angle.See Figure 1 for details.

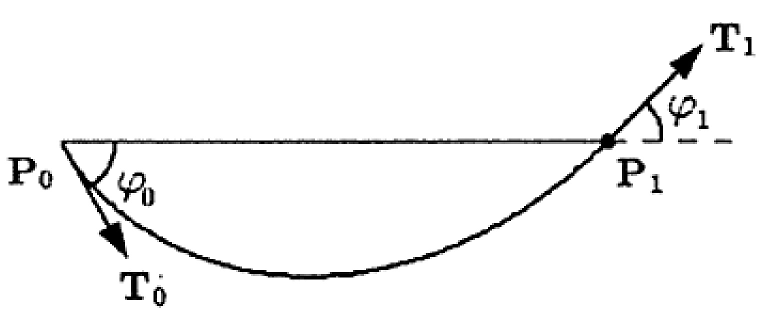

Figure 1 initial condition schematic diagram

Cubic Bézier curve can be expressed:

$$b(t)=\sum_{i=0}^{3} b_i B_i^3(t), t\in[0,1]. \tag{1}$$

the $B_i^3(t)=\binom{3}{i}t^i(1-t)^{3-i}$ is cubic Bernstein Polynomials.

The Bézier curves and endpoint properties available:

$$b_0 = P_0,\ b_3 = P_1. \tag{2}$$

Because the curve $b(t)$ satisfies G^1 geometric continuity at the end point, it can be obtained:

$$b_1 - b_0 = \frac{\beta_0}{3}T_0,\ b_3 - b_2 = \frac{\beta_1}{3}T_1.$$

Among them, β_0 and β_1 are parameters greater than zero.Take (2) into the upper simplification and get the following:

$$b_1 = P_0 + \frac{\beta_0}{3}T_0,\ b_2 = P_1 - \frac{\beta_1}{3}T_1. \tag{3}$$

So we can get a satisfied cubic Bézier continuous interpolation curve of all control points are $b_i, i = 0,1,2,3$. Generally speaking,parameter β_0, β_1 must be greater than zero,To meet the needs of all kinds of shape preserving interpolation in practical applications.So here is β_0, β_1 Feasible domain D:

$$D=\left\{(\beta_0,\beta_1)\in R^2 \middle| l_0 \le \frac{\beta_0}{\|P_1-P_0\|}\le u_0, l_1 \le \frac{\beta_1}{\|P_1-P_0\|}\le u_1\right\}, \tag{4}$$

Among this $u_i > l_i > 0, (i = 0,1)$.

Curvature smoothing usually means that the curvature changes very smoothly, that is, the rate of curvature change is minimal.This paper using the square of the three order derivative of Bézier curve that $\int (b'''(t))^2 dt$ in the definition of integral interval to curvature changes in the Bézier expression of zier curves on the gross.The minimum constraint problem based on curvature change is transformed to solve the minimum extreme problem. $\int (b'''(t))^2 dt$ that is

$$\min_{\beta_0,\beta_1} f = \int_0^1 (b'''(t))^2 dt$$

Due to the change of curvature b''' that is a nonlinear function, Therefore, it is very difficult to directly derive its explicit expression. To solve such mathematical problems, we usually use numerical integration approximation to function, and then get the equation solution. The Gauss-Legendre quadrature formula has high accuracy and fast convergence speed. Therefore, we apply two point Gauss-Legendre integral formula to deal with the objective function.

The Gauss quadrature formula is the interpolation quadrature formula with the highest algebraic accuracy.Through the nodes of the formula (5) $x_k \in [a,b], k = 0,1,\ldots,n$

And the coefficient of quadrature $A_k \geq 0, k = 0,1,\ldots,n$ Appropriate selection can make its algebraic accuracy the highest. Using the root of the last Legendre orthogonal polynomial of the special interval as the node, we can establish the Gauss-Legendre type formula (6).

$$\int_a^b f(x)dx \approx \sum_{k=0}^{n} A_x f(x_k), \tag{5}$$

$$\int_{-1}^{1} f(x)dx \approx f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right) \tag{6}$$

For the Gauss type quadrature formula (5), the remaining items are [12]

$$E(f) = \sum_a^b f(x)dx - \sum_{k=0}^{n} A_k f(x_k) = \int_a^b \frac{f^{(2n+2)}(\xi)}{(2n+2)!}\omega^2(x)dx, \xi \in [a,b] \tag{7}$$

Among $\omega(x) = \prod_{i=0}^{n}(x - x_i)$, x_i is the Gauss point.

In particular, when $n = 1$,(7)is a two point Gauss-Legendre formula

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\left(f\left(x\left(-\frac{\sqrt{3}}{3}\right)\right) + f\left(x\left(\frac{\sqrt{3}}{3}\right)\right)\right) \tag{8}$$

And $x(t) = \frac{a+b}{2} + \frac{b-a}{2}t$, the remaining item is

$$E(f) = \int_a^b \frac{f^{(4)}(\xi)}{4!}\left(x - x\left(-\frac{\sqrt{3}}{3}\right)\right)^2\left(x - x\left(\frac{\sqrt{3}}{3}\right)\right)^2 dx, \quad \xi \in [a,b] \tag{9}$$

By the two point Gauss-Legendre integral formula:

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\sum_{i=1}^{2} A_i f\left(\frac{a+b}{2} + \frac{b-a}{2}x_i\right), i = (1,2). \tag{10}$$

Look-up table and get $x_1 = -0.577503, x_2 = 0.5773503; A_1 = A_2 = 1.$

Then $\min_{\beta_0,\beta_1} f = \int_0^1 \left(b'''(t)\right)^2 dt$ simplification is as follows:

$$\min_{\beta_0,\beta_1} f \approx \frac{1}{2}\left\{\left[b'''(0.2112485)\right]^2 + \left[b'''(0.7887545)\right]^2\right\}$$

The b''' got by $$b^{(r)}(t) = \frac{n!}{(n-r)!}\sum_{r=0}^{n-r}\Delta^r b_i B_i^{n-r}(t), 1 \le r \le n. \quad (11)$$

$$\Delta^r b_i = \Delta^{r-1} b_{i+1} - \Delta^{r-1} b_i, \Delta b_i = b_{i+1} - b_i$$

The final result of the discrete results using the block coordinate descent method [13]The optimal solution in the feasible region D is obtained.

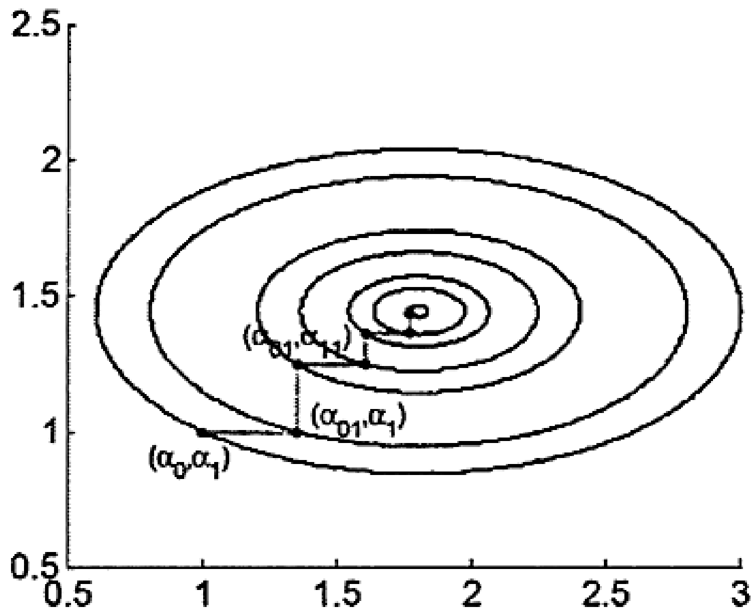

Figure 2 Schematic diagram of the principle of block coordinate descent method

4 Example verification and conclusion

By using Matlab[14]the algorithm is programmed as a running program, and data is tested at the same time.The result of this method is compared with the traditional curve with the minimum curvature change rate. It is proved that the interpolation curve based on the minimum curvature change rate constraint in this paper can get a better curve.For convenience of observation and calculation, set $P_0 = (0,0)$, $P_1 = (1,0)$.Figure 3 and 4 are Interpolation curves of 2 sets of restructures (RBS is Reconstructed Bézier Spline)，The left side is the interpolation curve, and the right side is the corresponding curvature map.

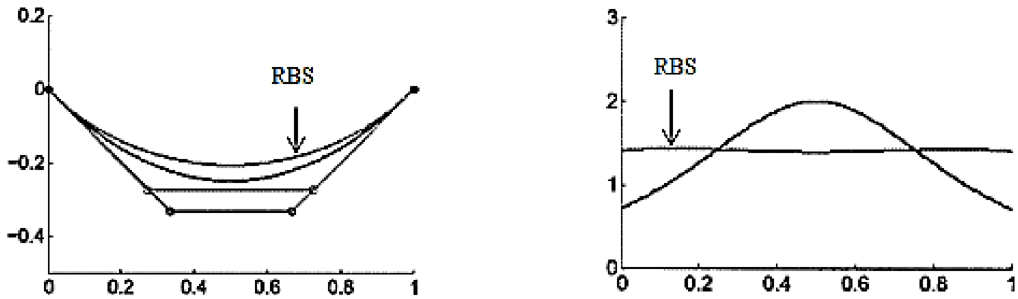

Figure 3 $\varphi_0 = \pi/4$, $\varphi_1 = \pi/4$

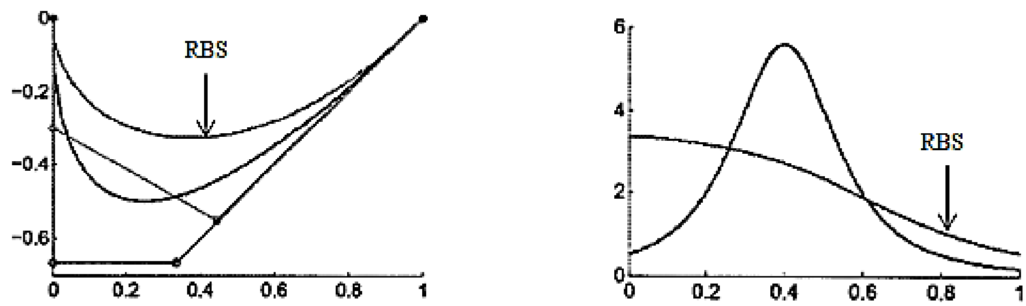

Figure 4 $\varphi_0 = \pi/2$, $\varphi_1 = \pi/4$

The curvature change of the right side shows that the curvature change of the interpolated curve using the curvature constraint is more gentle, which is more in line with the requirements of the smooth curve and meets the design requirements.

This paper describes a based on curvature change cubic interpolation Bézier curves and the minimum design method.In the whole curve construction process, the first Bézier curves model satisfies the initial condition of structure, In order to achieve the effect of smoothing, we choose curvature change rate integral as the objective function. Then, by optimizing the equation, we get the value of unknown parameter in the objective function, and get the curve with the smallest curvature change.Because the curvature change rate is used as a constraint in the solution process, the obtained curves not only have good interpolation effect, but also change the curvature more smoothly.

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